The tail is likewise curved up underneath, and lies with its broad surface towards the body, turning either towards the right or the left, and thickening part of the hinder extremities. In three examples the extremities are fully developed, and even show the characteristic discs on the tops of the toes. In the fourth example all four extremities present short stumps, and as yet show no traces of toes, whereas, as is well known, in the *Batrachia anura* generally the hinder extremities and the ends of the feet first appear. Neither of branchiæ nor of branchial slits is there any trace. On the other hand, in the last-mentioned example, the tail is remarkably larger, and has its broad surface closely adherent to the inner wall of the vesicle, and very full of vessels, so that there can be no doubt of its function as a breathing organ. As development progresses, the yelk-bag on the belly and the tail become gradually smaller, so that at last, when the little animal, being about 5 mill. long, bursts through the envelope, the tail is only 1·8 mill. in length, and after a few hours only 0·3 mill. long, and in the course of the same day becomes entirely absorbed. Examples of the same batch of ova, which were placed in spirit eight days after their birth, have a length of from 7·0 to 7·5 mill., whence we may conclude that their growth is not quicker than in other species of Batrachians.

The development of this frog, Dr. Peters observes (and probably of all the nearly allied species), without metamorphosis, without branchiæ, with contemporaneous evolution of the anterior and posterior extremities, as in the case of the higher vertebrates, and within a vesicle, like the amnion of these latter, if not strictly equivalent to it, is truly remarkable. But this kind of development is not quite unparalleled in the Batrachians, for it has long been known that the young of *Pipa americana* come forth from the eggs laid in the cells on their mother's back tailless and perfectly developed. In them, likewise, no one has yet detected branchiæ, and we also know from the observations of Camper,[1] that the embryos at an earlier period are provided with a tail-like appendage, which in this case also, may be perhaps regarded as an organ of breathing, possibly corresponding to the yelk-placenta of the hag-fish. As regards this point, also, Laurenti says of the *Pipa*: "Pulli ex loculamentis dorsi prodeuntes, metamorphosi nulla?" (Syn. Rept., p. 25.)

It would be of the highest interest, Dr. Peters adds, to follow exactly this remarkable development on the spot. The development of the embryo of these Batrachians in a way very like that of the scaled Reptilia makes one suspect that an examination of the temporary embryonic structures of *Hylodes* and *Pipa* would result in showing remarkable differences from those of other Batrachians. The general conclusions which might be drawn from this discovery are so obvious, says Dr. Peters, in conclusion, that it would be superfluous to put them forward.

A subsequent communication of Dr. Peters to the Academy informs us that it had escaped his notice that M. Bavay, of Guadaloupe, had already published some observations on the development of *Hylodes martinicensis*.[2] According to his observations, on each side of the heart there is a branchia consisting of one simple gill-arch, which on the seventh day is no longer discernible. On the ninth day there is no longer a trace of a tail, and on the tenth day the little animal emerges from the egg. M. Bavay also observed the contemporaneous development of the four extremities, and hints at the function of the tail as an organ of breathing.

The observations of Dr. Gundlach, therefore, says Dr. Peters, differ in some respects from those of M. Bavay. It would be specially desirable, however, to ascertain whether the arched vessel on each side of the heart is really to be regarded as a gill-arch, or only as the incipient bend of the aorta.

[1] Comm. Soc. Reg. Gotting. Cl. phys. ix p. 135 (1788).
[2] Ann. Sc. Nat. ser. 5, xvii., art. No. 16 (1873.)

TYPICAL LAWS OF HEREDITY[1]

WE are far too apt to regard common events as matters of course, and to accept many things as obvious truths which are not obvious truths at all, but present problems of much interest. The problem to which I am about to direct attention is one of these.

Why is it when we compare two groups of persons selected at random from the same race, but belonging to different generations of it, we find them to be closely alike? Such statistical differences as there may be, are always to be ascribed to differences in the general conditions of their lives; with these I am not concerned at present, but so far as regards the processes of heredity alone, the resemblance of consecutive generations is a fact common to all forms of life.

In each generation there will be tall and short individuals, heavy and light, strong and weak, dark and pale, yet the proportions of the innumerable grades in which these several characteristics occur tends to be constant. The records of geological history afford striking evidences of this. Fossil remains of plants and animals may be dug out of strata at such different levels that thousands of generations must have intervened between the periods in which they lived, yet in large samples of such fossils we seek in vain for peculiarities which will distinguish one generation taken as a whole from another, the different sizes, marks and variations of every kind, occurring with equal frequency in both. The processes of heredity are found to be so wonderfully balanced and their equilibrium to be so stable, that they concur in maintaining a perfect statistical resemblance so long as the external conditions remain unaltered.

If there be any who are inclined to say there is no wonder in the matter, because each individual tends to leave his like behind him, and therefore each generation must resemble the one preceding, I can assure them that they utterly misunderstand the case. Individuals do *not* equally tend to leave their like behind them, as will be seen best from an extreme illustration.

Let us then consider the family history of widely different groups; say of 100 men, the most gigantic of their race and time, and the same number of medium men. Giants marry much more rarely than medium men, and when they do marry they have but few children. It is a matter of history that the more remarkable giants have left no issue at all. Consequently the offspring of the 100 giants would be much fewer in number than those of the medium men. Again these few would, on the average, be of lower stature than their fathers for two reasons. First, their breed is almost sure to be diluted by marriage. Secondly, the progeny of all exceptional individuals tends to "revert" towards mediocrity. Consequently the children of the giant group would not only be very few but they would also be comparatively short. Even of these the taller ones would be the least likely to live. It is by no means the tallest men who best survive hardships, their circulation is apt to be languid and their constitution consumptive.

It is obvious from this that the 100 giants will not leave behind them their quota in the next generation. The 100 medium men, on the other hand, being more fertile, breeding more truly to their like, being better fitted to survive hardships, &c., will leave more than their proportionate share of progeny. This being so, it might be expected that there would be fewer giants and more medium-sized men in the second generation than in the first. Yet, as a matter of fact, the giants and medium-sized men will, in the second generation, be found in the same proportions as before. The question, then, is this:—How is it that although each individual does *not* as a rule leave his like behind him, yet successive generations resemble each other with great exactitude in all their general features?

[1] Lecture delivered at the Royal Institution, Friday evening, February 9, by Francis Galton, F.R.S.

It has, I believe, become more generally known than formerly, that although the characteristics of height, weight, strength, and fleetness are different things, and though different species of plants and animals exhibit every kind of diversity, yet the differences in height, weight, and every other characteristic, are universally distributed in fair conformity with a single law.

The phenomena with which it deals are like those perspectives spoken of by Shakespeare which, when viewed awry, show nothing but confusion.

Our ordinary way of looking at individual differences is awry; thus we naturally but wrongly judge of differences in stature by differences in heights, measured from the ground, whereas on changing our point of view to that whence the law of deviation regards them, by taking the average height of the race, and not the ground, as the point of reference, all confusion disappears, and uniformity prevails.

It was to Quetelet that we were first indebted for a knowledge of the fact that the amount and frequency of deviation from the average among members of the same race, in respect to each and every characteristic, tends to conform to the mathematical law of deviation.

The diagram contains extracts from some of the tables,

Scale of Heights.	American soldiers, 25,878 observations.		France (Hargenvilliers).		Belgium, Quetelet. 20 years' observations.	
Metres.	Observed	Calculated.	Observed	Calculated.	Observed	Calculated
1·90	1	3				
1·90	7	5				
87	14	13		1	1	1
84	25	28		3	2	3
81	45	52	25	7	7	7
79	99	84		16	14	14
76	112	117	32	32	34	28
73	138	142	55	55	48	53
70	148	150	88	87	102	107
68	137	137	114	118	138	136
65	93	109	144	140	129	150
62	109	75	140	145	162	150
60	49	45	116	132	106	136
57	14	24		105	110	107
54	8	11		73		53
51	1	4		44		28
48		1		24		14
45			286	11	147	7
42				4		3
39				2		1
36				1		
	1000	1000	1000	1000	1000	1000

Degrees of Dynanometer.	Lifting power of Belgian Men. Observed.	Calculated.
200	1	1
190		
180	9	10
170		
160	23	23
150		
140	32	32
130		
120	22	23
110		
100	12	10
90	1	1
	100	100

by which he corroborates his assertion. Three of the series in them refer to the heights of Americans, French, and Belgians respectively, and the fourth to strength, to that of Belgians. In each series there are two parallel columns, one entitled "observed," and the other "calculated," and the close conformity between each of the pairs is very striking.

These Tables serve another purpose; they enable those who have not had experience of such statistics to appreciate the beautiful balance of the processes of heredity in ensuring the repetition of such finely graduated proportions as those they record.

The outline of my problem of this evening is, that since the characteristics of all plants and animals tend to conform to the law of deviation, let us suppose a typical case, in which the conformity shall be exact, and which shall admit of discussion as a mathematical problem, and find what the laws of heredity must then be to enable *successive* generations to maintain statistical identity.

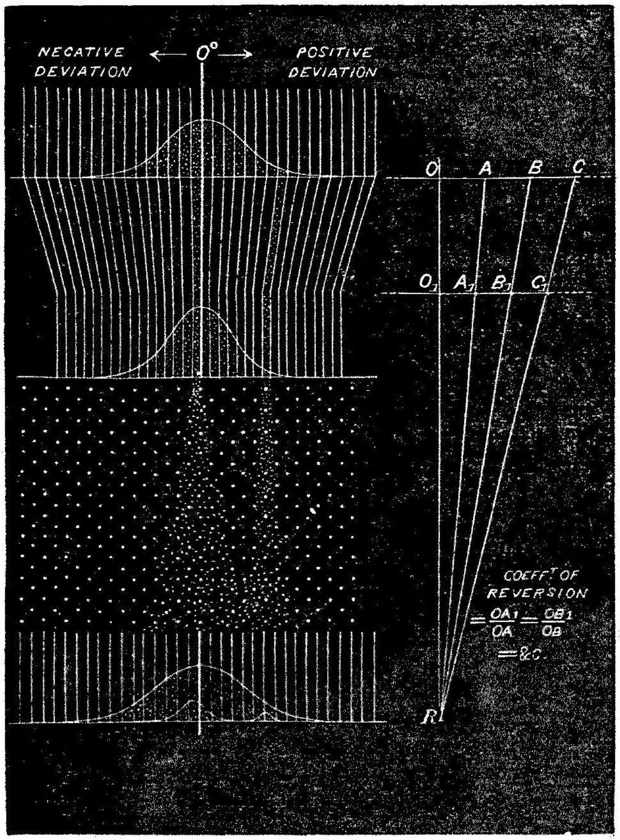

Fig. 1.

I shall have to speak so much about the law of deviation, that it is absolutely necessary to tax your attention for a few minutes to explain the principle on which it is based, what it is that it professes to show, and what the two numbers are which enable long series to be calculated like those in the tables just referred to. The simplest way of explaining the law is to begin by showing it in action. For this purpose I will use an apparatus that I employed three years ago in this very theatre, to illustrate other points connected with the law of deviation. An extension of its performance will prove of great service to us to-night, but I will begin by working the instrument as I did on the previous occasion. The portion of it that then existed and to which I desire now to confine your attention, is shown in the lower part of Fig. 1, where I wish you to notice the stream issuing from either of the divisions just above the dots, its dispersion among them, and the little heap that it forms on the bottom line. This part of the apparatus is like a harrow with its spikes facing us; below these are vertical compartments; the whole is faced with a glass plate. I will pour pellets from any point above the spikes, they will fall against the spikes, tumble about among them, and after pursuing devious paths, each will finally sink to rest in the compartment that lies beneath the place whence it emerges from its troubles.

The courses of the pellets are extremely irregular, it is rarely that any two pursue the same path from beginning to end, yet notwithstanding this you will observe the regularity of the outline of the heap formed by the accumulation of pellets.

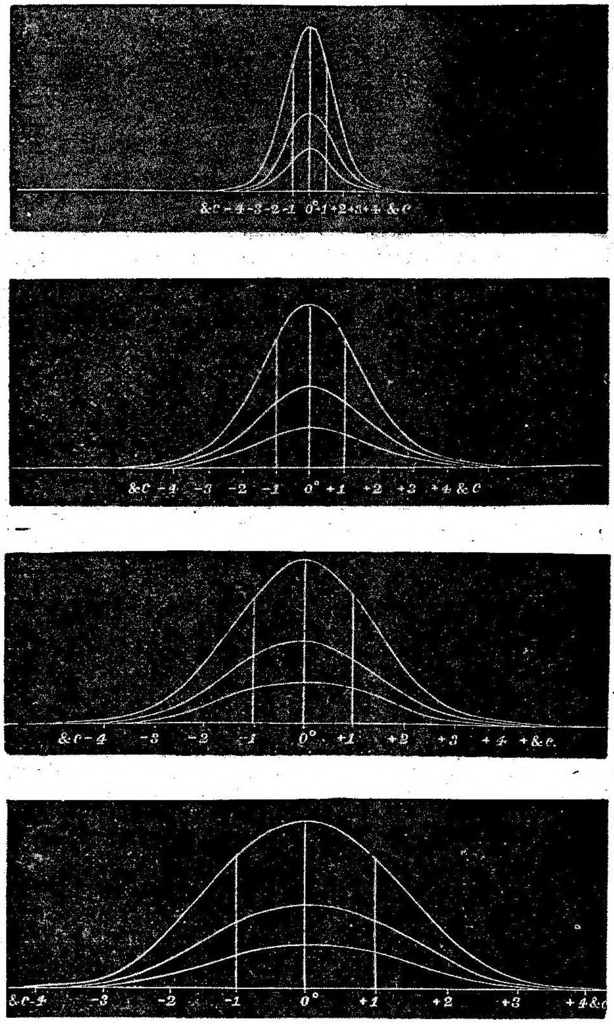

Fig. 2.

This outline is the geometrical representation of the curve of deviation. If the rows of spikes had been few, the deviation would have been slight, almost all the pellets would have lodged in a single compartment and would then have resembled a column; if they had been very numerous, they would have been scattered so widely that the part of the curve for a long distance to the right and left of the point whence they were dropped would have been of uniform width, like an horizontal bar. With intermediate numbers of rows of teeth, the curved contour of the heap would assume different shapes, all having a strong family resemblance. I have cut some of these out of cardboard; they are represented in the diagrams (Figs. 2 and 3). Theoretically speaking, every possible curve of deviation may be formed by an apparatus of this sort, by varying the length of the harrow and the number of pellets poured in. Or if I draw a curve on an elastic sheet of india-rubber, by stretching it laterally I produce the effects of increased dispersion; by stretching it vertically I produce that of increased numbers. The latter variation is shown by the successive curves in each of the diagrams, but it does not concern us to-night, as we are dealing with proportions, which are not affected by the size of the sample. To specify the variety of curve so far as dispersion is concerned, we must measure the amount of lateral stretch of the india-rubber sheet. The curve has no definite ends, so we have to select and define two points in its base, between which the stretch may be measured. One of these points is always taken directly below the place where the pellets were poured in. This is the point of no deviation, and represents the mean position of all the pellets, or the average of a race. It is marked as 0°. The other point is conveniently taken at the foot of the vertical line that divides either half of the symmetrical figure into two equal areas. I take a half curve in cardboard that I have again divided along this line, the weight of the two portions is equal. This distance is the value of 1° of deviation, appropriate to each curve.

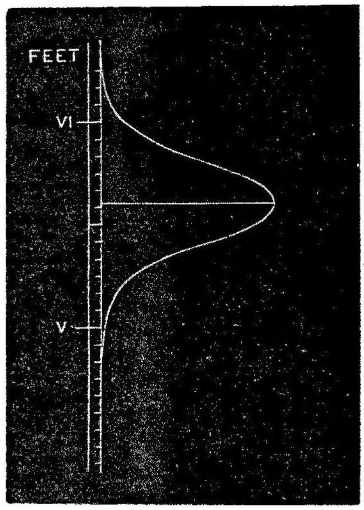

Fig. 3.

We extend the scale on either side of 0° to as many degrees as we like, and we reckon deviation as positive, or to be added to the average, on one side of the centre say to the right, and negative on the other, as shown in the diagrams. Owing to the construction, one quarter or 25 per cent. of the pellets will lie between 0° and 1°, and the law shows that 16 per cent. will lie between + 1° and + 2°, 6 per cent. between + 2° and + 3°, and so on. It is unnecessary to go more minutely into the figures, for it will be easily understood that a formula is capable of giving results to any minuteness and to any fraction of a degree.

Let us, for example, deal with the case of the American soldiers. I find, on referring to Gould's Book, that 1° of deviation was in their case 1·676 inches. The curve I hold in my hand has been drawn to that scale. I also find that their average height was 67·24 inches. I have here a standard marked with feet and inches. I apply the curve to the standard, and immediately we have a geometrical representation of the statistics of height of all those soldiers. The lengths of the ordinates show the proportion of men at and about their heights, and the area between any pairs of ordinates give the proportionate number of men between those limits.

It is indeed a strange fact that any one of us sitting quietly at his table could, on being told the two numbers just mentioned, draw out a curve on ruled paper, from which thousands of vertical lines might be chalked side by side on a wall, at the distance apart that is taken up by each man in a rank of American soldiers, and know that if the same number of these American soldiers taken indiscriminately had been sorted according to their heights and marched up to the wall, each man of them would find the chalked line which he found opposite to him to be of exactly his own height. So far as I can judge from the run of the figures in the table, the error would never exceed a quarter of an inch, except at either extremity of the series.

The principle of the law of deviation is very simple. The important influences that acted upon each pellet were the same; namely, the position of the point whence it was dropped, and the force of gravity. So far as these are concerned, every pellet would have pursued an identical path. But in addition to these there were a host of petty disturbing influences, represented by the spikes among which the pellets tumbled in all sorts of ways. The theory of combination shows that the commonest case is that where a pellet falls equally often to the right of a spike as to the left of it, and therefore drops into the compartment vertically below the point where it entered the harrow. It also shows that the cases are very rare of runs of luck carrying the pellet much oftener to one side than the other. The law of deviation is purely numerical; it does not regard the fact whether the objects treated of are pellets in an apparatus like this, or shots at a target, or games of chance, or any other of the numerous groups of occurrences to which it is or may be applied.[1]

I have now done with my description of the law. I know it has been tedious, but it is an extremely difficult topic to handle on an occasion like this. I trust the application of it will prove of more interest.

(*To be continued.*)

ON THE STRUCTURE AND ORIGIN OF METEORITES[2]

THE study of meteorites is naturally divisible into several very distinct branches of inquiry. Thus in the first place we may regard them as shooting stars, and observe and discuss their radiant points and their relation to the solar system. This may be called the astronomical aspect of the question. Then, when solid masses fall to the ground, we may study their chemical composition as a whole, or that of the separate mineral constituents; and lastly, we may study their mechanical structure, and apply to this investigation the same methods which have yielded such important results in the case of terrestrial rocks. So much has been written on the astronomical, chemical, and mineralogical aspect of my subject by those far more competent than myself to deal with such questions, that I shall confine my remarks almost entirely to the mechanical structure of meteorites and meteoric irons, and more especially to my own observations, since they will, at all events, have the merit of greater originality and novelty. Time will, however, not permit me to enter into the detail even of this single department of my subject.

In treating this question it appeared to me very desirable to exhibit to you accurate reproductions of the natural objects, and I have therefore had prepared photographs of my original drawings, which we shall endeavour to show by means of the oxyhydrogen lime-light, and I shall modify my lecture to meet the requirements of the case, exhibiting and describing special examples, rather than attempt to give an account of meteorites in general. Moreover, since the time at my disposal is short, and their external characters may be studied to great advantage at the British Museum, I shall confine my remarks as much as possible to their minute internal structure, which can be seen only by examining properly prepared sections with more or less high magnifying powers.

By far the greater part of my observations were made about a dozen years ago. I prepared a number of sections of meteorites, meteoric irons, and other objects which might throw light on the subject, and my very best thanks are due to Prof. Maskelyne for having most kindly allowed me to thoroughly examine the very excellent series of thin sections, which had been prepared for him. During the last ten years my attention has been directed to very different subjects, and I have done little more than collect material for the further and more complete study of meteorites. When I have fully utilised this material I have no doubt that I shall be able to make the subject far more complete, and may find it necessary to modify some of my conclusions. I cannot but feel that very much more remains to be learned, and I should not have attempted to give an account of what I have so far done, if I had not been particularly asked to do so by Mr. Lockyer. At the same time I trust that I shall at all events succeed in showing that the microscopical method of study yields such well marked and important facts, that in some cases the examination of only a single specimen serves to decide between rival theories.

In examining with the naked eye an entire or broken meteorite we see that the original external outline is very irregular, and that it is covered by a crust, usually, but not invariably black, comparatively thin, and quite unlike the main mass inside. This crust is usually dull, but sometimes, as in the Stannern meteorite, bright and shining, like a coating of black varnish. On examining with a microscope a thin section of the meteorite, cut perpendicular to this crust, we see that it is a true black glass filled with small bubbles, and that the contrast between it and the main mass of the meteorite is as complete as possible, and the junction between them sharply defined, except when portions have been injected a short distance between the crystals. We thus have a most complete proof of the conclusion that the black crust was due to the true igneous fusion of the surface under conditions which had little or no influence at a greater depth than $\frac{1}{100}$th of an inch. In the case of meteorites of different chemical composition, the black crust has not retained a true glassy character, and is sometimes $\frac{1}{10}$th of an inch in thickness, consisting of two very distinct layers, the internal showing particles of iron which have been neither melted nor oxidised, and the external showing that they have been oxidised and the oxide melted up with the surrounding stony matter. Taking everything into consideration, the microscopical structure of the crust agrees perfectly well with the explanation usually adopted, but rejected by some authors, that it was formed by the fusion of the external surface, and was due to the very rapid heating which takes place when a body moving with planetary velocity rushes into the earth's atmosphere—a heating so rapid that the surface is melted before the heat has time to penetrate beyond a very short distance into the interior of the mass.

When we come to examine the structure of the original interior part of meteorites, as shown by fractured surfaces, we may often see with the naked eye that they are mottled in such a way as to have many of the characters of a brecciated rock, made up of fragments subsequently cemented together and consolidated. Mere rough fractures are, however, very misleading. A much more accurate opinion may be formed from the examination of a smooth flat surface. Facts thus observed led Reichenbach to conclude that meteorites had been formed by the

[1] Quetelet, apparently from habit rather than theory, always adopted the binomial law of error, basing his tables on a binomial of high power. It is absolutely necessary to the theory of the present paper, to get rid of binomial limitations and to consider the law of deviation or error, in its exponential form.

[2] Abstract of lecture delivered by H. C. Sorby, F.R.S., &c., at the Museum, South Kensington, on March 10.

which conjugates, and which is furnished with only two cilia. The only distinction between the macro- and microzoospores seems to be that the former have four cilia, the latter only two. When the microzoospores fail to conjugate they may develop non-sexually just like the macrozoospores. This is a fact of the highest importance. In this plant, belonging to the lowest group in which sexual reproduction occurs, the sexual and non-sexual zoospores are hardly to be distinguished, and if by any chance union of the sexual zoospores does not take place, the zoospore behaves like a macrozoospore and develops non-sexually.

After remaining in a state of rest, sometimes for nearly twelve months, the contents of the zygospore break up into zoospores, from which arise the filamentous stage of Ulothrix.

In Ulothrix the conjugating cells are generally morphologically and physiologically identical; but sometimes larger zoospores conjugate with smaller, a difference in sex being here indicated. In other cases the microzoospores which have not conjugated germinate and give rise to individuals capable of reproducing. The study of the formation and subsequent development of the zygospore shows that the product of conjugation is to be considered as a new sexually-produced generation. It is a unicellular plantlet, with a root-like process and a slowly-growing plant-body which performs the function of assimilation. It in fact represents the embryo and the sporophore of the Pteridophytes. The root-end of the plantlet is formed by the union of the germinal spots of the conjugating microzoospores, while the assimilating plant-body represents the united chlorophyll-bearing parts of the zoospore.

The Ulothrix is thus one of the Zygosporeæ, and is probably related to Hydrodictyon, but it shows certain affinities to Sphæroplea, the lowest of the Oosporeæ.

As this part concludes the tenth volume of this serial, a most useful table of contents and special index of names of plants and details treated of in all the papers in the ten volumes has been added by Herr Zopf. This enables the student at once to refer to any given plant, or even to the part of it described in the various papers.

W. R. McNab

THE ROYAL NAVAL COLLEGE, GREENWICH

On February 1, 1873, the Royal Naval College was opened at Greenwich, "for the purpose of providing for the education of naval officers of all ranks above that of midshipman in all branches of theoretical and scientific study bearing upon their profession." The first annual report on the Royal Naval College thus established has been recently presented to both Houses of Parliament.

When the College was established it was determined by the Admiralty to bring together in it all the necessary means both for the higher education of naval officers and also of others connected with the navy. During the session which terminated last year four captains, four commanders, ninety-three lieutenants, and eight navigating-lieutenants joined the college as students, but of these only one captain, thirty-three lieutenants, and three navigating-lieutenants went through the whole nine months' course, although one captain, two commanders, fifty lieutenants, and three navigating-lieutenants underwent the final examination. Besides these officers, who may all be regarded as being purely voluntary students, there was also a large number of others studying at the college, with a view to passing certain examinations, which would qualify them either for promotion or advancement or for appointment to some special branch or department of the service.

Finally, ten private students are reported as having passed through a course of instruction, nine of the number being foreign officers, a fact which testifies to the estimation in which the college is held abroad.

With regard to the subjects of study we find that, besides the course of mathematics, which is compulsory for all students, systematic courses of instruction, extending over the entire session, are given in physics, chemistry, steam, navigation, and nautical astronomy, marine surveying, permanent and field fortification, military surveying and drawing, military history, foreign languages—namely, French, German, and Spanish—and in freehand drawing. Special courses of lectures are also given on various subjects, among which the principal seem to be the Structural Arrangements of Men-of-War, International Law, Naval History, and Practical Ship-building.

TYPICAL LAWS OF HEREDITY[1]

II.

First let me point out a fact which Quetelet and all writers who have followed in his paths have unaccountably overlooked, and which has an intimate bearing on our work to-night. It is that, although characteristics of plants and animals conform to the law, the reason of their doing so is as yet totally unexplained. The essence of the law is that differences should be wholly due to the collective actions of a host of independent *petty* influences in various combinations, as was represented by the teeth of the harrow, among which the pellets tumbled in various ways. Now the processes of heredity that limit the number of the children of one class such as giants, that diminish their resemblance to their fathers, and kill many of them, are not petty influences, but very important ones. Any selective tendency is ruin to the law of deviation, yet among the processes of heredity there is the large influence of natural selection. The conclusion is of the greatest importance to our problem. It is, that the processes of heredity must work harmoniously with the law of deviation, and be themselves in some sense conformable to it. Each of the processes must show this conformity separately, quite irrespectively of the rest. It is not an admissible hypothesis that any two or more of them, such as reversion and natural selection, should follow laws so exactly inverse to one another that the one should reform what the other had deformed, because characteristics, in which the relative importance of the various processes is very different, are none the less capable of conforming closely to the typical condition.

When the idea first occurred to me, it became evident that the problem might be solved by the aid of a very moderate amount of experiment. The properties of the law of deviation are not numerous and they are very peculiar. All, therefore, that was needed from experiment was suggestion. I did not want proof, because the theoretical exigencies of the problem would afford that. What I wanted was to be started in the right direction.

I will now allude to my experiments. I cast about for some time to find a population possessed of some measurable characteristic that conformed fairly well to the law, and that was suitable for investigation. I determined to take seeds and their weights, and after many preparatory inquiries, fixed upon those of sweet-peas. They were particularly well suited to my purposes; they do not cross-fertilise, which is a very exceptional condition; they are hardy, prolific, of a convenient size to handle, and their weight does not alter when the air is damp or dry. The little pea at the end of the pod, so characteristic of ordinary peas, is absent in sweet peas. I weighed seeds individually, by thousands, and treated them as a census officer would treat a large population. Then I selected with great pains several sets for planting. Each set contained seven little packets, and in each packet were ten seeds, precisely of the same weight. Number one of the packets contained giant seeds, all as nearly as might be of $+3^{\circ}$ of deviation. Number seven contained very

[1] Lecture delivered at the Royal Institution, Friday evening, February 9, by Francis Galton, F.R.S. Continued from p. 495.

small seeds, all of -3° of deviation. The intermediate packets corresponded severally to the intermediate degrees $\pm 2^\circ \pm 1^\circ$ and 0°. As the seeds are too small to exhibit, I have cut out discs of paper in strict proportion to their sizes, and strips in strict proportion to their weights, and have hung below them the foliage produced by one complete set. Many friends and acquaintances each undertook the planting and culture of a complete set, so that I had simultaneous experiments going on in various parts of the United Kingdom. Two proved failures, but the final result was this: that I obtained the more or less complete produce of seven sets, that is of $7 \times 7 \times 10$, or 490 carefully weighed seeds.

It would be wholly out of place if I were to enter into the details of the experiments themselves, the numerous little difficulties and imperfections in them, or how I balanced doubtful cases, how I divided returns into groups, to see if they confirmed one another, or how I conducted any other of the well-known statistical operations. Suffice it to say that I took immense pains, which if I had understood the general conditions of the problem as clearly as I do now, I should not perhaps have cared to bestow. The results were most satisfactory. They gave me two data, which were all that I required in order to understand the simplest form of descent, and so I got at the heart of the problem at once.

Simple descent means this. The parentage must be single, as in the case of the sweet peas which were not cross-fertilised, and the rate of production and the incidence of natural selection must both be independent of the characteristic. The processes concerned in simple descent are those of Family Variability and Reversion. It is well to define these words clearly. By family variability is meant the departure of the children of the same or similarly descended families from the ideal mean type of all of them. Reversion is the tendency of that ideal mean type to depart from the parent type, "reverting" towards what may be roughly and perhaps fairly described as the average ancestral type. If family variability had been the only process in simple descent, the dispersion of the race would indefinitely increase with the number of the generations, but reversion checks this increase, and brings it to a standstill, under conditions which will now be explained.

On weighing and sorting large samples of the produce of each of the seven different classes of the peas, I found in every case the law of deviation to prevail, and in every case the value of 1° of deviation to be the same. I was certainly astonished to find the family variability of the produce of the little seeds to be equal to that of the big ones, but so it was, and I thankfully accept the fact, for if it had been otherwise I cannot imagine, from theoretical considerations, how the problem could be solved.

The next great fact was that Reversion followed the simplest possible law; for the proportion was constant between the deviation of the mean weight of the produce generally and the deviation of the parent seed, reckoning in every case from one standard point. In a typical case, that standard must be the mean of the race, otherwise the deviation would become unsymmetrical, and cease to conform to the law.

I have adjusted an apparatus (Fig. 1) to exhibit the action of these two processes. We may consider them to act not simultaneously but in succession, and it is purely a matter of convenience which of the two we suppose to act the first. I suppose first Reversion then Family Variability. That is to say, I suppose the parent first to revert and then to tend to breed his like. So there are three stages: (1) the population of parents, (2) that of reverted parents, (3) that of their offspring. In arranging the apparatus I have supposed the population to continue uniform in numbers. This is a matter of no theoretical concern, as the whole of this memoir relates to the distinguishing peculiarities of samples irrespectively of the absolute number of individuals in those samples. The apparatus consists of a row of vertical compartments, with trap-doors below them, to hold pellets which serve as representatives of a population of seeds. I will begin with showing how it expresses Reversion. In the upper stage of the apparatus the number of pellets in each compartment represents the relative number in a *population* of seeds, whose weight deviates from the average, within the limits expressed by the distances of the sides of that compartment from the middle point. The correct shape of the heap has been ensured by my having cut a slit of the proper curvature in the board that forms the back of the apparatus. As it is glazed in front I have only to pour pellets in from above until they reach the level of the slit. Such overplus as may have been poured in will run through the slit, to waste, at the back. The pellets to the right of the heap represent the heaviest seeds, those to the left the lightest. I shall shortly open the trap-door on which the few representatives of the giant seeds rest. They will run downwards through an inclined shoot, and fall into another compartment nearer the centre than before. I shall repeat the process on a second compartment in the upper stage, and successively on all the others. Every shoot converges towards one standard point in the middle vertical line; thus the present shape of the heap of pellets is more contracted in width than it was before, and is of course more humped up in the middle. We need not regard the humping up; what we have to observe is that each degree of deviation is simultaneously lessened. The effect is as though the curve of the first heap had been copied on a stretched sheet of india-rubber that was subsequently released. It is obvious from this that the process of reversion co-operates with the general law of deviation. Fig. 6 shows the principle of the process of reversion clearly.

I have now to exhibit the effects of variability among members of the same family. It will be recollected that the produce of peas of the same class deviated normally on either side of their own mean weight; that is to say, I must make the pellets which were in each of the upper compartments to deviate on either side of the compartment in which they now lie, which corresponds to that of the medium weight of their produce. I open the trap-door below one of the compartments in the second stage, the pellets run downwards through the harrow, dispersing as they run, and form a little heap in the lowest compartments, the centre of which heap lies vertically below the trap-door through which they fell. This is the contribution to the succeeding generation of all the individuals belonging to the compartment in the upper stage from which they came. They first reverted and then dispersed. I open another trap-door, and a similar process is gone through; a few extreme pellets in this case add themselves to the first formed heap. Again, I continue the process; heap adds itself to heap, and when all the pellets have fallen through, we see that the aggregate contributions bear an exact resemblance to the heap from which we originally started. A simple formula (see Appendix) expresses the conditions of equilibrium. I attended to these, when I cut out the slit in the back board of the upper compartment, by which the shape of the original heap was regulated. Thus it follows from the formula that if deviation after reversion was to deviation before reversion as 4 to 5, and if 1° of family variability was six units, then the value of 1° in the population must be ten units.

It is easy to prove that the bottom heap is strictly a curve of deviation, and that its scale tends invariably to become the same as that of the upper one. It will be recollected that I showed that every variety of curve of deviation was producible by variations in the length of the harrow, and that if the pellets were intercepted at successive stages of their descent they would form a succession of curves of increasing scales of deviation. The curve in the second stage may therefore be looked upon as one of these intercepts; all that it receives in sinking to the third stage being an additional dose of dispersion.

As regards the precise scale of deviation that characterises each population, let us trace, in imagination, the history of the descendants of a single medium-sized seed. In the first generation the differences are merely those due to family variability; in the second generation the tendency to wider dispersion is somewhat restrained by the effect of reversion; in the third, the dispersion again increases, but is more largely restrained, and the same process continues in successive generations, until the step-by-step progress of dispersion has been overtaken and exactly checked by the growing antagonism of reversion. Reversion acts precisely after the law of an elastic spring, as was well shown by the illustration of the india-rubber sheet. Its tendency to recoil increases the more it is stretched, hence equilibrium must at length ensue between reversion and family variability, and therefore the scale of deviation of the lower heap must after many generations always become identical with that of the upper one.

We have now surmounted the greatest difficulty of our problem; what remains will be shortly disposed of. This refers to sexual selection, productiveness, and natural selection. Let us henceforth suppose the heights and every other characteristic of all members of a population to be reduced to a uniform adult male standard, so that we may treat it as a single group. Suppose, for example, a female whose height was equal to the average female height $+ 3°$ of female deviation, the equivalent in terms of male stature is the average male height $+ 3°$ of male deviation. Hence the female in question must be registered not in the feet and inches of her actual height, but in those of the equivalent male stature.

On this supposition we may take the numerical mean of the stature of each couple as the equivalent of a single parent, so that a male parent plant having 1° deviation and a female parent plant having 2° of deviation, would together rank as a single fertilised plant $0 + 1\frac{1}{2}°$.

In order that the law of sexual selection should co-operate with the conditions of a typical population, it is necessary that selection should be *nil*, that is, that there should not be the least tendency for tall men to marry tall women rather than short ones. Each strictly typical quality taken by itself must go for nothing in sexual selection. Under these circumstances one of the best known properties of the law of deviation (technically called that of "two fallible measures") shows that the population of sums of couples would conform truly to the law, and the value of 1° would be that of the original population multiplied by $\sqrt{2}$. Consequently the population of *means of couples* would equally conform to the law, but in this case the 1° of original deviation would have to be divided by $\sqrt{2}$, the deviations of means of couples being half that of sums of couples.

The two remaining processes are productiveness and survival. Physiologically they are alike, and it is reasonable to expect the same general law to govern both. Natural selection is measured by the percentage of survival among individuals born with like characteristics. Productiveness is measured by the average number of children from all parents who have like characteristics, but it may physiologically be looked upon as the percentage of survival of a vast and unknown number of possible embryos, producible by such parents. The number being unknown creates no difficulty if they may be considered to be, on the average, the same in every class. Experiment could tell me little about either natural selection or productiveness. What I have to say is based on plain theory. I can explain this best by the process of natural selection. In each species, the height, &c., the most favoured by natural selection, is the one in which the demerits of excess or deficiency are most frequently balanced. It is therefore not unreasonable to look at nature as a marksman, her aim being subject to the same law of deviation as that which causes the shot on a target to be dispersed on either side of the point aimed at. It would not be difficult, but it would be tedious, to justify the analogy; however, it is unnecessary to do so, as I propose to base the analogy on the exigences of the typical formula, no other supposition being capable of fulfilling its requirements. Suppose for a moment that nature aims, as a marksman, at the medium class, on purpose to destroy and not to save it. Let a block of stone (Fig. 4)

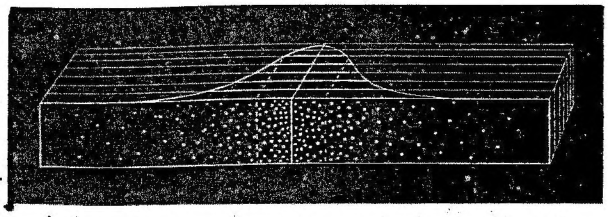

Fig. 4.

represent a rampart, and let a gun be directed at a vertical line on its side on purpose to breach it: the shots would fall with the greatest frequency in the neighbourhood of the vertical line, and their marks would diminish in frequency as the distance increased, in conformity with the law of deviation. Each shot batters away a bit of stone, and the shape of the breach would be such that its horizontal outline will be the well-known curve. This would be the action of nature were she to aim at the destruction of medium sizes. Her action as preserver of them is the exact converse, and would be represented by a cast that filled the gap and exactly replaced the material that had been battered away. The percentage of thickness of wall that had been destroyed at each degree of deviation is represented by the ordinate of the curve, therefore the percentage of survival is also an ordinate of the same curve of deviation. Its scale has a special value in each instance, subject to the general condition in every typical case, that its 0° shall correspond to the 0° of deviation of height, or whatever the characteristic may be.

In Fig. 5 the thickness of wall that has been destroyed at each degree of deviation is represented by the corresponding ordinate of the horizontal outline of the portion which remains. Similarly, in the case of an original

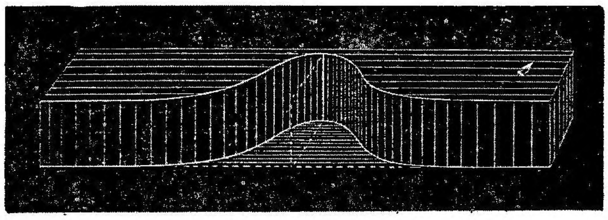

Fig. 5.

population, in which each class was equally numerous, the amount of survivors at each degree of deviation is also represented by the corresponding ordinate of this or a similar curve.

But in the original population at which we are supposing nature to aim the representatives of each class are not equally numerous, but are arranged according to the law of deviation; the middle class being most numerous, while the extreme classes are but scantily represented. The ordinate of the above-mentioned outline will in this case represent, not the *absolute number*, but the *percentage* of survivors at each degree of deviation.

(*To be continued.*)

TYPICAL LAWS OF HEREDITY[1]

III.

If a graphic representation is desired, which will give the absolute number of survivors at each degree, we must shape the rampart which forms nature's target so as to be highest in the middle and to slope away at each side according to the law of deviation. Thus Fig. 6 represents the curved rampart before it has been aimed at; Fig. 7, afterwards.

I have taken a block of wood similar to Fig. 4, to represent the rampart; it is of equal height throughout. A cut has been made at right angles to its base with a fret-saw, to divide it in two portions—that which would remain after it had been breached, Fig. 5, and the cast of the breach. Then a second cut with the fret-saw has been made at right angles to its face, to cut out of the rampart an equivalent to the heap of pellets that represents the original population. The gap that would be made in the heap and the cast that would fill the gap are curved on two faces, as in the model. This is sufficiently represented in Fig. 7.

The operation of natural selection on a population already arranged according to the law of deviation is represented more completely in an apparatus, Fig. 8, which I will set to work immediately.

It is faced with a sheet of glass. The heap, as shown in the upper compartment of the apparatus, is three inches in thickness, and the pellets rest on slides. Directly below the slides, and running from side to side of the apparatus, is a curved partition, which will separate the pellets as they fall upon it, into two portions, one that runs to waste at the back, and another that falls to the front, and forms a new heap. The curve of the partition is a curve of deviation. The shape of this heap is identical with the cast of the gap in Fig. 7. It is highest and thickest in the middle, and it fines away towards either extremity. When the slide upon which it rests is removed, the pellets run down an inclined plane that directs them into a frame of uniform and shallow depth. The pellets from the deep central compartments (it has been impossible to represent in the diagram as many of these as there were in the apparatus) will stand very high from the bottom of the shallow frame, while those that came from the distant compartments will stand even lower than they did before. It follows that the selected pellets form, in the lower compartment, a heap of which the scale of deviation is much more contracted than that of the heap from which it was derived. It is perfectly normal in shape, owing to an interesting theoretical property of deviation (see formulæ at end of this memoir).

Productiveness follows the same general law as survival, being a percentage of possible production, though it is usual to look on it as a simple multiple, without dividing by the 100. In this case the front face of each compartment in the upper heap represents the number of the parents of the same class, and the depth of the partition below that compartment represents the average number that each individual of that class produces.

To sum up. We now see clearly the way in which the resemblance of a population is maintained. In the purely typical case, each of the processes of heredity and selection is subject to a well-defined and simple law, which I have formulated in the appendix. It follows that when we know the values of 1° in the several curves of family variability, productiveness, and survival, and when we know the co-efficient of reversion, we know absolutely all about the ways in which that characteristic will be distributed among the population at large.

I have confined myself in this explanation to purely typical cases, but it is easy to understand how the actions of the processes would be modified in those that were not typical. Reversion might not be directed towards the mean of the race, neither productiveness nor survival might be greatest in the medium classes, and none of their laws may be strictly of the typical character. However, in all cases the general principles would be the same. Again, the same actions that restrain variability would restrain the departure of average values beyond certain limits. The typical laws are those which most nearly express what takes place in nature generally; they may never be exactly correct in any one case, but at the same time they will always be approximately true and always serviceable for explanation. We estimate through their means the effects of the laws of sexual selection, of productiveness, and of survival, in aiding that of reversion in bridling the dispersive effect of family variability. They show us that natural selection does not act by carving out each new generation according to a definite pattern on a Procrustean bed, irrespective of waste. They also explain how small a contribution is made to future generations by those who deviate widely from the mean, either in excess or deficiency, and enable us to calculate whence the deficiency of exceptional types is supplied. We see by them that the ordinary genealogical course of a race consists in a constant outgrowth from its centre, a constant dying away at its margins, and a tendency of the scanty remnants of all exceptional stock to revert to that mediocrity, whence the majority of their ancestors originally sprang.

Appendix.

I will now proceed to formulate the typical laws. In what has been written, 1° of deviation has been taken equal to the "probable error" = C × 0·4769 in the well-known formula $y = \frac{1}{c\sqrt{\pi}} . e^{-\frac{x^2}{c^2}}$. According to this, if x = amount of deviation in feet, inches, or any other external unit of measurement, then the number of individuals in any sample who deviate between x and $x + \delta x$ will vary as $e^{-\frac{x^2}{c^2}} \delta x$ (it will be borne in mind that we are for the most part not concerned with the coefficient in the above formula).

Let the modulus of deviation (c) in the original population, after the process has been gone through, of converting the measurements of all its members (in respect to the characteristic in question), to the adult male standard, be written c_0.

1. Sexual selection has been taken as *nil*, therefore the population of "parentages" is a population of which each unit consists of the mean of a couple taken indiscriminately. This, as well known, will conform to the law of deviation, and its modulus which we will write c_1 has already been shown to be equal to $\frac{1}{\sqrt{2}} . c_0$.

2. Reversion is expressed by a fractional coefficient of the deviation, which we will write r. In the "reverted" parentages (a phrase whose meaning and object have already been explained)

$$y = \frac{1}{rc\sqrt{\pi}} . e^{-\frac{x^2}{r^2 c^2}}.$$

In short, the population, of which each unit is a reverted parentage, follows the law of deviation, and has its modulus, which we will write c_2, equal to rc_1.

3. Productiveness :—We saw that it followed the law of deviation; let its modulus be written f. Then the number of children to each parentage that differs by the amount of x from the mean of the parentages generally (*i.e.*, from the mean of the race), will vary as $e^{-\frac{x^2}{f^2}}$; but the number of such parentages varies as $e^{-\frac{x^2}{c_2^2}}$, therefore if each child

[1] Lecture delivered at the Royal Institution, Friday evening, February 9, by Francis Galton, F.R.S. Continued from p. 514.

absolutely resembled his parent, the number of children who deviated x would vary as $e^{-\frac{x^2}{f^2}} \times e^{-\frac{x^2}{c_2^2}}$, or as $e^{-x^2\left\{\frac{1}{f^2}+\frac{1}{c_2^2}\right\}}$. Hence the deviations of the children in their amount and frequency would conform to the law, and the modulus of the population of children in the supposed case of absolute resemblance to their parents, which we will write c_3, is such that—

$$\frac{1}{c_3} = \sqrt{\left(\frac{1}{f^2}+\frac{1}{c^2}\right)}.$$

We may, however, consider the parents to be multiplied

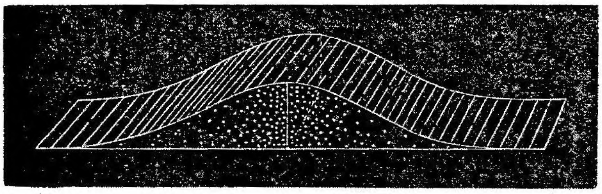

FIG. 6.

and the productivity of each of them to be uniform. It is more convenient than the converse supposition and it comes to the same thing. So we will suppose the reverted parentages to be more numerous but equally prolific, in which case their modulus will be c_3, as above.

4. Family variability was shown by experiment to follow the law of deviation, its modulus, which we will write v, being the same for all classes. Therefore the amount of deviation of any one of the offspring from the mean of his race is due to the combination of two influences, the deviation of his "reverted" parentage and his own family variability; both of which follow the law of deviation. This is obviously an instance of the well-known law of the "sum of two fallible measures" (Airy, "Theory of Errors," § 43). Therefore the modulus of the population in the present stage, which we will write c_4, is equal to $\sqrt{(v^2 + c_3^2)}$.

5. Natural selection follows, as has been explained, the same general law as productiveness. Let its modulus be written s; then the percentage of survivals among children, who deviate x from the mean, varies as $e^{-\frac{x^2}{s^2}}$, and for the same reasons as those already given, its effect will be to leave the population still in conformity

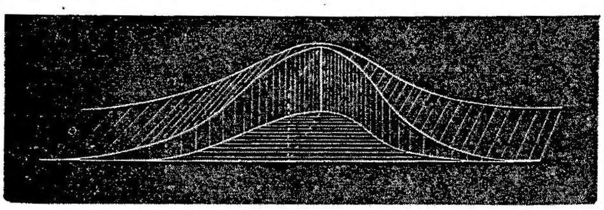

FIG. 7.

with the law of deviation, but with an altered modulus, which we will write c_5, and

$$\frac{1}{c_5} = \sqrt{\left(\frac{1}{s^2}+\frac{1}{c_4^2}\right)}.$$

Putting these together we have, starting with the original population having a modulus $= c_0$:—

1. $c_1 = \frac{1}{\sqrt{2}} c_0$.

2. $c_2 = rc_1$.

3. $c_3 = \sqrt{\left\{\frac{f^2 c_2^2}{f^2 + c_2^2}\right\}}$.

4. $c_4 = \sqrt{\left\{v^2 + c_3^2\right\}}$.

5. $c_5 = \sqrt{\left\{\frac{s^2 c_4^2}{s^2 + c_4^2}\right\}}$.

And lastly, as the condition of maintenance of statistical resemblance in consecutive generations:—

6. $c_5 = c_0$.

Hence, given the coefficient r and the moduli v, f, s, the value of c_0 (or c_5) can be easily calculated.

In the case of simple descent, which was the one first

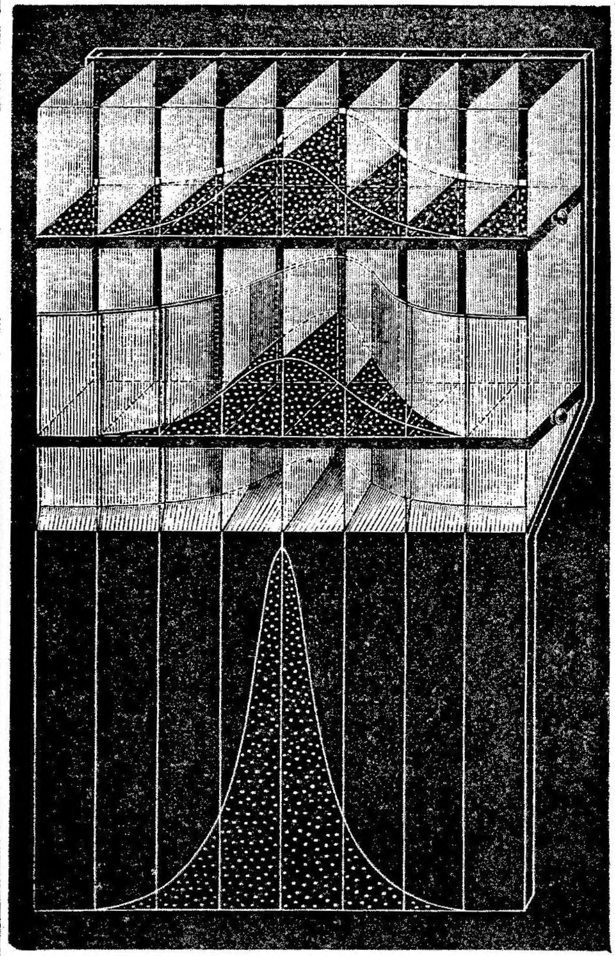

FIG. 8.

considered, we have nothing to do with c_0, but begin from c_1. Again, as both fertility and natural selection are in this case uniform, the values of f and s are infinite. Consequently our equations are reduced to—

$$c_2 = rc_1\,;\ c_4 = \sqrt{\left\{v^2 + c_2^2\right\}}\,;\ c_4 = c_1,$$

whence

$$c_1^2 = \frac{v^2}{1 - r^2}.$$

CARL FRIEDRICH GAUSS

BORN APRIL 30, 1777, DIED FEBRUARY 23, 1855.[1]

DE MORGAN in his "Budget of Paradoxes" (p. 187), tells the following story:—The late Francis Baily wrote a singular book, "Account of the Rev. John Flamsteed, the first Astronomer-Royal:" it was published by the Admiralty for distribution, and the author drew up the distribution list.

[1] We adopt the date given by the Baron Sartorius von Waltershausen in his "Gauss. Zum Gedächtniss," Leipzig, 1856. Encyclopedists and other authorities are pretty equally divided between this date and April 23. All the English Cyclopædias we have consulted, with the exception of Chambers's (1874), give April 23. We may also mention that on the list of students at the Collegium Carolinum the name is Johann Friedrich Karl Gauss. We have followed Gauss himself in our heading.